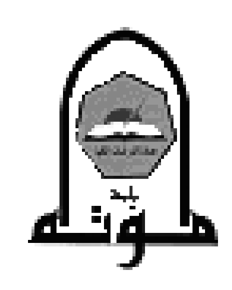


Mu'tah University

College of Graduate studies

On The Uniform Approximation by a Holomorphic Functions in One Complex Variable

By:

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A Thesis Submitted to the College of the Graduate Studies in Partial Fulfillment of the Requirements for the Degree of Master of Science, Department of Mathematics and Statistics

Mu'tah University, 2015

Dedication

To my Parents.

To my husband and my children.

To my brothers and my sister.

Acknowledgement

Praise to Allah. The most gracious and merciful for giving me the ability to achieve this work. Special appreciation goes to my supervisor, Prof Rateb Al-Btoush for his supervision, guidance, help and encouragement during the preparation of this thesis and throughout my years of study. I also thank him for setting very high standards for me in research as well as in writing.I hope healthy and happiness for him and his family.

I am grateful to my parents and my husband for their unlimited encouragement and their sacrifices during my life. I wish them all happiness and my God please them.

Esra -Al rawashdeh

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Abstract

On Uniform Approximation by Holomorphic Functions in One Complex Variable

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In this thesis some properties and conditions of uniform approximation of functions of complex variables in a compact subset of complex numbers have been studied and verified. Such properties and conditions have been used to ensure the existence and uniqueness of best approximation by holomorphic functions. In addition to that extra results are obtained regarding Mergelian and Runge's approximation theorems for functions by polynomials and rational functions on a compact subset of complex numbers.

الملخص

حول التقريب المنتظم باستخدام الاقترانات التحليلية بمتغير مركب واحد

إسراء زياد الرواشدة

جامعة مؤتة، 2015

في هذه الأطروحة تمت دراسة بعض خواص وشروط التقريب المنتظم للدالة المركبة التحليلية على مجموعة جزئية متراصة من الأعداد المركبة. لقد تم استخدام هذه الشروط والخواص للتأكد من وحدانية ووجود أفضل تقريب للدالة المركبة بدالة مركبة تحليلية. علاوة على ذلك تم الحصول على بعض النتائج الإضافية المتعلقة بنظرية ميرقيليان ونظرية رونق للتقريب باستخدام كثيرات الحدود والدوال الكسرية على مجموعة جزئية متراصة من الأعداد المركبة.

List of Symbols

$\mathbb{C}$: Set of complex numbers.

$\mathfrak{J}_A$: Topology for A.

$Y \Subset X$: Y is relatively compact in X.

K°: Set of interior points of K.

K^c: Complement of K.

$\mathfrak{T}(Y)$: Vector space of all infinitely differentiable functions on Y.

$\mathfrak{T}^{0,1}(Y)$: Subspace of $\mathfrak{T}(Y)$of differential forms of type (0,1).

$\Omega(Y)$: Vector space of holomorphic 1- forms.

$\mathrm{Supp}(f)$: Support of f.

$H(K)$: Class of all holomorphic functions on K.

$f|Y$: The restriction of function f to set Y.

$\wedge$: Exterior product of complex vector spaces.

C(K): Collection of all continuous complex functions on K.

$C_c(X)$: Collection of all continuous complex functions on X whose support is compact.

$C_c^{\infty}(X)$: Collection of all infinitely differentiable complex functions on Xwhose support is compact.

$P_V(f)$: Best approximation to f from V.

$A(K)$: Class of all complex continuous function on K that are analytic in the interior of K.

$d\mu$: Finite measure supported on K.

$\mathrm{K}^{+}$and K^{-}: Positive and negative parts of K.

u.a.p: Uniformly approximated by polynomials.

$E(f)$: Error of approximation of f.

$\overline{K}$: Closure of K.

$\mathcal{A}$: A cover of set A.

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